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# Social status and crime

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## Abstract

We consider a large population of agents choosing either to engage in a criminal activity or working. Individuals feel varying degrees of self-reproach if they commit criminal acts. In addition, they are concerned with their social status in society, based on others' perceptions of their values. In making their decisions, individuals weigh both the material and social risks of being a criminal and a worker. We find that introducing social status concerns may induce multiple equilibria. We also consider the implications of intragroup and intergroup interactions in an economy with two classes of earning abilities. Typically, there is more crime in the low ability group and increasing punishment reduces crime, but the opposite may also be true.

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## 1. Introduction

Until recently, economic analysis of crime has focused only on the material risks associated with illegal activities. Although many researchers are quick to note that internal motives such as guilt, virtue, shame, and moral values are also influential in criminal behavior, most work has focused on how extrinsic incentives deter or encourage criminal activities. The decision to focus only on material interests may be justified by the fact that incentives such as the probability of arrest, the severity of punishment, earning ability, etc., are much easier to measure and identify than internal motives. Moreover, if the variation of crime rates across time and space were attributable to a large extent to differences in sentence lengths, police expenditures, and other relevant socio-economic factors, there would be little need to discuss unobservable internal motives that have little impact on individuals' decisions.

Recent empirical research, however, has shown that observable attributes can explain only a small portion of the wide variation of crime. In this paper, we identify two related internal motives. We presume that individuals are born with a publicly unobservable propensity to feel self-reproach, or guilt, after committing an illegal act.<sup>1</sup> For our purposes, these sentiments determine an individual's personal ethical and moral values. In addition, we presume that individuals are also concerned about how they are perceived by others.<sup>2</sup> Being perceived as an individual with a low level of guilt is embarrassing or shameful and thus induces a loss of social status. We interpret these two motives as an individual's private and public types, the latter being a function of her observable actions. Therefore, in our model an individual with a low propensity of guilt may nevertheless choose not to be engaged in illegal activities in order to maintain her social status, or to avoid embarrassment. In other words, if individuals are sufficiently concerned about their social status, they may be inclined to act "as if" they are moral.

Our discussion is based on the seminal paper by Bernheim [3] and Bénabou and Tirole [2], in which actions signal an individual's personal attributes and therefore affect her status in the society.<sup>3</sup> Unlike Bernheim [3], however, we

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<sup>1</sup>See Kaplow and Shavell [8] for a model that considers the optimal use of guilt and virtue in deterring crime. Also, see Conley and Wang [4] for a model in which agents differ from one another with respect to their exogenously given levels of earning ability and ethical value.

<sup>2</sup>Our discussion is similar to the psychological games framework introduced by Geanakoplos, Pearce, and Stacchetti [6].

<sup>3</sup>Alternatively, one could imagine that social status is directly dependent on individual's (observable) actions. See, for example, the model of Lindbeck, Nyberg, and Weibull [9], in

introduce an additional complexity. Whether or not an individual has actually engaged in a criminal act is private information and therefore does not determine an individual's status. Instead, in our context, an individual's status depends on the publicly observable event of whether the individual was arrested or not.

Our approach is quite similar to Rasmusen's [10] model of crime, in which a convicted criminal suffers welfare losses not only as a consequence of material penalties but also from stigmatization, i.e. others' reluctance to interact with the individual. Rasmusen [10] also considers conviction as the basis for being stigmatized. However, in our model, the personal cost of being arrested is not the economic opportunities foregone for that person, but rather the embarrassment of being perceived as an immoral individual. We find that introducing social status concerns may induce multiple equilibria.

The extent to which an individual suffers public embarrassment also depends on the class structure of the society and the way that social status is determined within that structure. Consider, for example, a society in which individuals are grouped according to their earning abilities. If criminal opportunities are identical among the classes, then one would expect that crime would be more prevalent in lower ability groups. However, if social status is determined within each group, then one would expect that being arrested yields less social status loss in low ability groups than in high ability groups.

Several papers have distinguished between the implications of local and aggregate social interaction. In particular, Gleaser, Sacerdote, and Scheinkman [5] consider a model where agents influence or are influenced by the actions of their neighbors. The authors find that these social interactions create enough covariance across individuals to explain the high variance of crime rates among different neighborhoods. In particular, the authors find that social interactions are quite high in petty crimes, such as property crimes, larceny, etc. Also Sah [11] discusses a model in which individuals formulate their likelihood of arrest by interacting locally. Our approach is quite distinct from these models, however, since we consider rational individuals whose internal motives are shaped partially by their beliefs about their social status. Allowing two groups with different abilities, we find that when the social interaction is aggregate, there will be more criminals that belong

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which welfare recipients incur costs of being observed as living on transfers. Note, however, that such an approach may be a reduced form of Bernheim's [3] model. In particular, being observed as a welfare recipient may be interpreted as a signal of an underlying attribute. For example, individuals may be concerned with their own ability to support themselves, which, in turn, may be related to their earning ability, taste for leisure, or other fundamentals.

to the low ability group. However, when there is only local interaction within a group, there may be more crime in the high ability group than the low ability group. Also, we find that there are some cases in which increasing punishment increases crime.

In the next section, we introduce a simple activity choice model and show that concern for social status loss induces multiple equilibria. In section 3, we introduce an aggregate relationship between work and crime. In section 4, we allow the society to be comprised of two ability groups with a large number of agents within each group and discuss the implications of intragroup and intergroup interaction. Section 5 concludes the paper.

## 2. Simple model of crime

Consider an infinite population of agents. The population size is normalized to 1. Agents simultaneously decide whether to engage in an illegal activity ( $c = 1$ ) or a legal activity ( $c = 0$ ). Let  $U_c \in \mathcal{R}$  denote an individual's utility from activity  $c$ . In order to simplify our discussion, in the rest of the paper we assume that utilities are in monetary terms and the agents are risk neutral. Individuals differ from one another with respect to the psychic cost/benefit of engaging in the illegal activity, which is denoted by  $\gamma \in \mathcal{R}$ .

Suppose that  $\gamma$  is distributed by a continuous unimodal probability density function  $\phi$  with its mean and median at zero. Individuals with  $\gamma > 0$  feel more self-reproach, remorse, or guilt, than the average person in the population when they engage in criminal activity. Individuals with  $\gamma < 0$ , however, may derive some pleasure from the act of crime. In what follows, we assume that while  $\gamma$  is private information, its distribution,  $\phi$ , is public information. Also, let  $\phi(\gamma) > 0$  for all  $\gamma \in \mathcal{R}$ .

If a criminal is arrested, she receives a punishment which costs her  $-f < 0$ . We presume that the probability of arrest is given by a constant likelihood,  $\alpha$ .<sup>4</sup> Also, suppose that only criminals are arrested. With these points in mind, individual  $\gamma$  will choose the legal activity if and only if

$$U_0 \geq U_1 - \alpha f - \gamma. \quad (2.1)$$

The equilibrium of the model will be described by the behavior of the marginal person, that is, by

$$\Gamma \equiv U_1 - U_0 - \alpha f, \quad (2.2)$$

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<sup>4</sup>Throughout the paper, we will assume that an arrest results in conviction.

which will serve as a threshold that separates criminals from non-criminals. In particular, all individuals below the threshold ( $\gamma < \Gamma$ ) will choose to be criminals and all individuals above the threshold ( $\gamma \geq \Gamma$ ) will choose to engage in the legal activity.<sup>5</sup> Also, the supply of crime, or the population proportion of criminals, can readily be rewritten as a function of the threshold  $\Gamma$ ,

$$K = \int_{-\infty}^{\Gamma} \phi(\gamma) d\gamma. \quad (2.3)$$

In the model described above, an optimal decision is finding the activity that yields the greatest utility, keeping in mind the risks involved in committing an illegal act. Our discussion thus far is an elementary version of Becker's [1] criminal choice model. In particular, for any  $\Gamma > 0$ , increasing punishment  $f$  or arrest rate  $\alpha$  will always reduce participation in illegal activities.

Individuals, however, may also be concerned with the stigma associated with becoming a criminal. As in Bernheim [3], we assume that status depends on the publicly perceived type of an individual, denoted by  $\mu \in \mathcal{R}$ , which is not necessarily equivalent to the agent's actual type  $\gamma$ . Unlike Bernheim's [3] approach, however, the public forms its perception about a specific agent based on the only observation available to them: Whether the individual was caught or not.

In order to simplify our discussion, we suppose that the social status value,  $s$ , of being perceived as a type  $\mu$  individual is simply a linear function of the individual's concern for her social status,  $z \geq 0$ , or

$$s(\mu) = \mu z. \quad (2.4)$$

In this social setting, if an individual is observed to have a low level of guilt,  $\mu < 0$ , then she will incur a status loss,  $s < 0$ . We assume that  $z$  is constant and identical for all individuals. Now, define  $\mu^a$  and  $\mu^n$  as the two perceived types based on the two events of being arrested and not being arrested, respectively. In order to ease notation, let  $s^a = s(\mu^a)$  and  $s^n = s(\mu^n)$ .

It is important to emphasize the difference between social status and guilt. Social status is not a personal attribute. It only describes an individual's condition based on how she is perceived by others, which can be interpreted as being embarrassed or feeling shameful. At best,  $\mu$  is a perceived state, dependent on observable outcomes. Guilt, on the other hand, is a personal attribute, which is privately known and exogenously given.

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<sup>5</sup>We assume that an individual will choose the legal activity if  $\gamma = \Gamma$ . Assuming otherwise would not alter any of our results.

With the social setting just described, individual  $\gamma$  will choose to engage in the legal activity if her extended utility from activity 0,

$$U_0 + s^n,$$

is no less than her extended utility from the illegal activity,

$$U_1 + (1 - \alpha) s^n - \alpha (f - s^a) - \gamma.$$

After rearranging, we see that an individual will choose to work if

$$U_0 \geq U_1 - \alpha (f + S) - \gamma,$$

where

$$S = s^n - s^a$$

denotes the net expected status loss of being arrested. Thus, a positive value for  $S$  would imply that being arrested is costly, though we do not impose any restrictions on the sign of  $S$ .

Each individual's action depends on the social status, which in turn depends on the behavior of others. As before, we can determine the threshold  $\Gamma$  which separates criminals from others in equilibrium. Also, since only criminals are arrested, the expected type of an arrested individual must simply be the average type for criminals, or

$$\mu^a = \frac{\int_{-\infty}^{\Gamma} \gamma \phi(\gamma) d\gamma}{K}. \quad (2.5)$$

Note that  $\mu^a < 0$  and  $\mu^a < \Gamma$  for all  $\Gamma$ .

The expected type of a "seemingly innocent" individual, however, must consider both non-criminals and successful criminals. Then, a random pick from the population of individuals who were not arrested may either be a non-criminal, which occurs with probability

$$\frac{1 - K}{1 - \alpha K},$$

or a successful criminal, which occurs with probability

$$\frac{K(1 - \alpha)}{1 - \alpha K}.$$

We can thus write the expected type of an individual not arrested as

$$\begin{aligned}\mu^n &= \frac{K(1-\alpha)}{1-\alpha K} \cdot \frac{\int_{-\infty}^{\Gamma} \gamma \phi(\gamma) d\gamma}{K} \\ &\quad + \frac{1-K}{1-\alpha K} \cdot \frac{\int_{\Gamma}^{\infty} \gamma \phi(\gamma) d\gamma}{1-K}\end{aligned}\tag{2.6}$$

$$= -\frac{\alpha K}{1-\alpha K} \cdot \mu^a,\tag{2.7}$$

where the last equality follows from the fact that the mean of  $\phi$  is zero. By rearranging and simplifying terms, we may now write the net expected status loss as

$$\begin{aligned}S &= (\mu^n - \mu^a) \cdot z \\ &= -\frac{\mu^a}{1-\alpha K} \cdot z > 0,\end{aligned}\tag{2.8}$$

implying that being arrested is costly.

The equilibrium condition is thus

$$W(\Gamma) \equiv U_0 - U_1 + \alpha(f + S) + \Gamma = 0.\tag{2.9}$$

Our next result shows that for some given distribution, there may be multiple equilibria for the simple model of criminal activity we have just described.

**Proposition 2.1.** *There may be multiple equilibria. In particular, there exist multiple equilibria for some distribution function  $\phi$  and social status concern  $z > 0$  if  $U_1 - U_0 - \alpha f \leq G$ .*

**Proof.** Note that there exists some small  $M_- \in \mathcal{R}$  such that for all  $\gamma < M_-$ ,  $W(\gamma) < 0$ . Similarly, it is easy to show that there exists some large  $M_+ \in \mathcal{R}$  such that  $W(\gamma) > 0$  for all  $\gamma > M_+$ . Therefore, if  $W(\gamma) = 0$  then we must have  $W_\Gamma = dW/d\Gamma > 0$  at equilibrium  $\Gamma$ .<sup>6</sup> Otherwise, there must be multiple equilibria.

We now show that  $W_\Gamma$  may be negative for some  $\phi$  and  $(\alpha, f)$ . First, note that

$$\mu_\Gamma^a = \frac{\Gamma - \mu^a}{K} \cdot \phi(\Gamma) > 0.$$

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<sup>6</sup>In what follows, a subscript  $\Gamma$  denotes a partial derivative with respect to  $\Gamma$ .



Noting that  $K_\Gamma = \phi(\Gamma)$ , we write the partial derivative of  $W$  for  $\Gamma$  as

$$\begin{aligned} W_\Gamma &= 1 + \alpha S_\Gamma \\ &= 1 - \left( \frac{\mu_\Gamma^a (1 - \alpha K) + \alpha \mu^a \phi(\Gamma)}{(1 - \alpha K)^2} \right) \cdot \alpha z \\ &= 1 + \left( \frac{\frac{\mu^a - \Gamma}{K} (1 - \alpha K) - \alpha \mu^a}{(1 - \alpha K)^2} \right) \cdot \phi(\Gamma) \alpha z \end{aligned} \quad (2.10)$$

$$= 1 + \left( \frac{(1 - 2\alpha K) \mu^a - (1 - \alpha K) \Gamma}{(1 - \alpha K)^2} \right) \cdot \frac{\phi(\Gamma)}{K} \cdot \alpha z. \quad (2.11)$$

We first show that when  $K < 1/2$ , the following inequality is correct:

$$(1 - 2\alpha K) \mu^a - (1 - \alpha K) \Gamma < 0. \quad (2.12)$$

Note that when the distribution  $\phi$  is truncated to the right at  $\Gamma < 0$ , the mean of the truncated distribution, which is  $\mu^a$ , must be less than the median of that distribution, implying that

$$\int_{\mu^a}^{\Gamma} \phi(\gamma) d\gamma > \frac{K}{2}.$$

Also, since  $\phi$  is increasing for all  $\Gamma < 0$ , it is clearly the case that

$$(\Gamma - \mu^a) \phi(\Gamma) = \int_{\mu^a}^{\Gamma} \phi(\Gamma) d\gamma \geq \int_{\mu^a}^{\Gamma} \phi(\gamma) d\gamma.$$

We then have

$$(\mu^a - \Gamma) \phi(\Gamma) \leq - \int_{\mu^a}^{\Gamma} \phi(\gamma) d\gamma \leq -\frac{K}{2},$$

or

$$\mu^a \leq -\frac{K}{2\phi(\Gamma)} + \Gamma. \quad (2.13)$$

Similarly,

$$-\Gamma \phi(\Gamma) = \int_{\Gamma}^0 \phi(\Gamma) d\gamma \leq \int_{\Gamma}^0 \phi(\gamma) d\gamma = \frac{1}{2} - K = \frac{1 - 2K}{2}. \quad (2.14)$$

We can now substitute (2.13) and (2.14) into equation (2.12) to get,

$$(1 - 2\alpha K) \mu^a - (1 - \alpha K) \Gamma \leq (1 - 2\alpha K) \left( -\frac{K}{2\phi(\Gamma)} + \Gamma \right) - (1 - \alpha K) \Gamma$$

$$\begin{aligned}
&= -\alpha K \Gamma - \frac{K}{2\phi(\Gamma)} \\
&= \frac{K}{2\phi(\Gamma)} \cdot (-2\alpha\phi(\Gamma)\Gamma - 1) \\
&\leq \frac{K}{2\phi(\Gamma)} \cdot (-\alpha(1 - 2K) - 1) < 0.
\end{aligned}$$

The above argument can be used to show that if  $\alpha < 1$ , then the (2.12) is correct when  $K = 1/2$ . Therefore, there exists some upper bound  $G > 0$  such that

$$(1 - 2\alpha K)\mu^a - (1 - \alpha K)\Gamma \leq 0,$$

for all  $\Gamma \leq G$ . Using equation (2.9), if  $z > 0$  and  $U_1 - U_0 - \alpha f \leq G$ , then  $\Gamma \leq G$ . ■

Having multiple equilibria diminishes our predictive power regarding the equilibrium outcomes. It is, however, possible to reduce the number of equilibria since not all equilibria are stable. Moreover, focusing on stable outcomes, we characterize the sensitivity of our model to the policy variables. We first introduce our notion of stability and then apply it to analyze the sensitivity of the model.

If  $\Gamma$  is locally stable, then a small perturbation of the threshold around  $\Gamma$  should initiate an adjustment which will end at  $\Gamma$ . Let us introduce  $t \geq 0$  as time and  $\Gamma(t)$  as the process which converges to  $\Gamma$ . We assume that the solution path is governed by the non-linear differential equation

$$\dot{\Gamma}(t) = \frac{d\Gamma(t)}{dt} \equiv -k \cdot W(\Gamma(t)), \quad (2.15)$$

for some arbitrary speed of adjustment  $k > 0$ . In words, if individual  $\gamma = \Gamma(t)$  would like to be a worker (criminal), or if  $W(\gamma) > 0$  ( $W(\gamma) < 0$ ), then equation (2.15) requires that  $\dot{\Gamma}(t) < 0$  ( $\dot{\Gamma}(t) > 0$ ).

**Lemma 2.2.** *The equilibrium is locally stable if and only if  $W_\Gamma = dW/d\Gamma > 0$ .*

**Proof.** We use a linear approximation of equilibrium dynamics to show that our condition is necessary and sufficient to imply that equilibrium  $\Gamma$  is locally stable. First, taking the linear approximation of equation (2.15) around  $\Gamma$ , we get

$$\dot{\Gamma}(t) = -k \cdot W_\Gamma(\Gamma) \cdot (\Gamma(t) - \Gamma)$$

for  $\Gamma(t)$  close to  $\Gamma$ . The previous equation and our concept of stability then imply that  $\dot{\Gamma}$  and  $(\Gamma(t) - \Gamma)$  have to have opposite signs. Therefore,  $W_\Gamma(\Gamma) > 0$  has to be true for local stability to be satisfied at  $\Gamma$ . ■

The next result shows that crime varies negatively with the policy parameters  $(\alpha, f)$  or the social concern  $z$ .

**Proposition 2.3.** *At a locally stable equilibrium, increasing the probability of arrest,  $\alpha$ , punishment,  $f$ , or social status concern,  $z$ , reduces crime.*

**Proof.** Let  $W$  be defined as above in equation (2.9). Stability implies that  $W_\Gamma > 0$  at any equilibrium. Implicitly differentiating equation (2.9), we have

$$\frac{d\Gamma}{df} = -\frac{W_f}{W_\Gamma}.$$

We know that  $W_f = \alpha \geq 0$ . Then, at any stable equilibrium,  $d\Gamma/df \leq 0$ , which proves our result. To obtain the results for  $\alpha$  and  $z$ , note that  $W_\alpha = f + S > 0$  and  $W_z = \alpha(\mu^n - \mu^a) > 0$ . ■

### 3. Property crimes

In the previous section, we discussed a simple model of crime without considering the aggregate relationship between crime and work. However, just as a criminal's earnings are uncertain whenever the probability of arrest is greater than zero, a worker's income is also uncertain simply because she may be robbed. Moreover, having more criminals implies that there will be fewer workers and, therefore, less income to be stolen. In this section, we investigate the aggregate relationship between criminal and work activities within a model of social status.

Let  $y \geq 0$  denote the income that a worker can earn. In this section, we assume that all individuals have the same earning ability, or that  $y$  is identical for all agents. If she chooses to be a worker, an agent is required to commit a constant amount of her time to work, which induces the psychic cost of  $g \geq 0$ , assumed to be common to all agents. We presume that engaging in the criminal activity requires no effort. Alternatively, we could assume that  $g$  gives the net cost of working instead of being a criminal. As before, the individuals have personal costs associated with crime, such as self-reproach, guilt, etc.

Criminal opportunities arrive randomly, as in İmrohoroglu, Merlo, and Rupert [7]. In order to simplify matters, suppose that each criminal is randomly assigned to another individual, implying that a criminal is unable to target her victim,

and that a criminal may have the opportunity to rob at most a single worker.<sup>7</sup> In addition, let us presume that no two criminals can both be assigned to the same worker. Crime occurs only if a criminal victimizes a worker. Then, for a criminal, the probability of being assigned to a worker is equivalent to the population proportion of workers,  $1 - K$ , where  $K$ , the population proportion of potential criminals, is defined as

$$K = \int_{-\infty}^{+\infty} c(\gamma) \phi(\gamma) d\gamma, \quad (3.1)$$

where  $c(\gamma)$  denotes the criminal choice of individual  $\gamma$ . Also, the population proportion of criminals that are able to victimize a worker will be  $K(1 - K)$ . Then, the probability that a worker will be victimized equals to the population proportion of criminals who victimize a worker divided by the population proportion of workers, or  $K(1 - K) / (1 - K) = K$ .

If a criminal is assigned to some random worker, then she will attempt to steal her entire income,  $y$ . Victimizing a worker does not guarantee that the criminal activity results in a successful robbery. In our model, a criminal, and only a criminal, can be caught. Again, let  $\alpha$  be the probability that a criminal – who has been able to victimize a worker – will be arrested. When a criminal is caught, we assume that the victim is returned the entire stolen amount and the criminal is charged a fine of  $f \geq 0$ . For a worker, the probability of being unharmed by crime is given by the probability of not being victimized by a successful criminal,  $1 - K(1 - \alpha)$ . Similarly, the probability that a criminal will be successful is equal to the joint probability that the criminal will be assigned to a worker and that the crime is a success,  $(1 - K)(1 - \alpha)$ . Let

$$\begin{aligned} U_0 &= (1 - K(1 - \alpha))y \\ U_1 &= (1 - K)(1 - \alpha)y \end{aligned}$$

denote an individual's expected income from work and crime, respectively.

In order to illustrate the choice problem, let us imagine the two activities as lotteries. Working results in three different outcomes. The worker can lose her income if a criminal is assigned to her and if the crime is a success, which occurs with probability  $(1 - \alpha)K$ . Otherwise, if either the criminal is caught or if she

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<sup>7</sup>A more complex model would also include opportunities to rob criminals, opportunities to form organizations that specialize in criminal activities, etc. We do not consider these options in this paper.

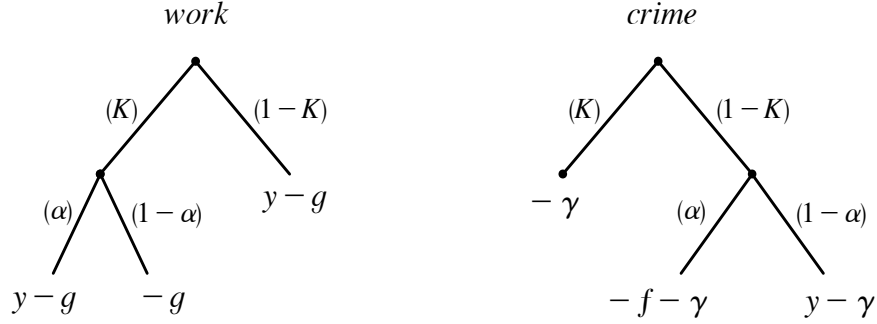


Figure 3.1: The two actions represented as lotteries.

is not victimized, the worker will be able to enjoy her entire income  $y$ . Engaging in crime also leads to three outcomes. As discussed above, the criminal is able to steal  $y$  if she is assigned to a worker and if the crime is successful. If she is not assigned to a worker, then the criminal will earn nothing. Instead, if the criminal is caught stealing, which occurs with probability  $(1 - K)\alpha$ , then she will return the entire stolen amount<sup>8</sup> and incur the punishment of  $f$ . The two decision trees in Figure 3.1 depict the work and crime activities as lotteries.

With these in mind, an individual will choose to be a worker if her expected utility from work,  $U_0 - g + s^n$  exceeds her expected utility from criminal activities,  $U_1 - \alpha(1 - K)(f - s^a) + (1 - \alpha(1 - K))s^n - \gamma$ . After rearranging, we can rewrite type  $\gamma$  individual's condition to work as

$$\alpha y - g \geq -\alpha(1 - K)(f + S) - \gamma, \quad (3.2)$$

where  $S = (\mu^n - \mu^a)z$  represents the net expected social status loss of being apprehended.

As before, we can determine the threshold  $\Gamma$  which separates criminals from others in equilibrium. The expected type of an arrested individual must simply be the average type for criminals,  $\mu^a$ , as defined in equation (2.5). An individual who is not arrested, however, may either be from the population of workers or from the population of criminals who are not arrested. The proportion of workers among all individuals who are not arrested is given by the proportion of workers,

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<sup>8</sup>Alternative formulations regarding whether a part of the stolen amount can be looted are possible but do not change our result.

$1 - K$ , divided by  $1 - \alpha K (1 - K)$ , the population proportion of all individuals who are not arrested. Similarly, the proportion of criminals among all individuals who are not arrested is given by the population proportion of criminals who are not arrested,  $K (1 - \alpha (1 - K))$ , divided again by  $1 - \alpha K (1 - K)$ . We can thus write the expected type of an individual who is not arrested as

$$\begin{aligned}\mu^n &= \frac{K (1 - \alpha (1 - K))}{1 - \alpha K (1 - K)} \cdot \frac{\int_{-\infty}^{\Gamma} \gamma \phi(\gamma) d\gamma}{K} \\ &\quad + \frac{1 - K}{1 - \alpha K (1 - K)} \cdot \frac{\int_{\Gamma}^{+\infty} \gamma \phi(\gamma) d\gamma}{1 - K}, \\ &= -\frac{\alpha K (1 - K)}{1 - \alpha K (1 - K)} \cdot \mu^a > 0.\end{aligned}\tag{3.3}$$

After rearranging, the net expected social loss  $S$  can be simplified to

$$S = -\frac{\mu^a}{1 - \alpha K (1 - K)} \cdot z \geq 0.\tag{3.4}$$

Social status risk is closely related to the signaling capacity of the event of being arrested. If the criminal is in an environment where many individuals with above-average guilt ( $\gamma \geq 0$ ) are criminals, then the social status loss associated with being caught will not be very dire.

**Lemma 3.1.** *When threshold  $\Gamma$  increases, the expected status loss decreases, or  $S_{\Gamma} < 0$ .*

**Proof.** The derivative of  $S$  with respect to  $\Gamma$  is

$$S_{\Gamma} = -\frac{(1 - \alpha K (1 - K)) \mu_{\Gamma}^a + \alpha (1 - 2K) \mu^a K_{\Gamma}}{(1 - \alpha K (1 - K))^2} z,\tag{3.5}$$

where

$$\mu_{\Gamma}^a = \frac{\Gamma K - \int_{-\infty}^{\Gamma} \gamma \phi(\gamma) d\gamma}{K^2} \phi(\Gamma) = \left( \frac{\Gamma - \mu^a}{K} \right) \phi(\Gamma) > 0.$$

The last inequality follows from the fact that  $\Gamma > \mu^a$  by definition. Substituting for  $\mu_{\Gamma}^a$  in (3.5), we can write

$$\begin{aligned}S_{\Gamma} &= -\frac{(1 - \alpha K (1 - K)) \left( \frac{\Gamma - \mu^a}{K} \right) + \alpha (1 - 2K) \mu^a}{(1 - \alpha K (1 - K))^2} \cdot \phi(\Gamma) z \\ &= \frac{(1 - \alpha K (2 - 3K)) \mu^a - (1 - \alpha K (1 - K)) \Gamma}{K (1 - \alpha K (1 - K))^2} \cdot \phi(\Gamma) z \leq 0\end{aligned}$$

The last inequality follows from the fact that

$$(1 - \alpha K(2 - 3K)) \mu^a - (1 - \alpha K(1 - K)) \Gamma < 0. \quad (3.6)$$

To see this, note that for  $K = 1/2$  it is an immediate consequence of  $\Gamma > \mu^a$ . Also, for  $K > 1/2$ ,  $\Gamma > 0$ , which implies that  $(1 - \alpha K(1 - K)) \Gamma > 0$ , while  $(1 - \alpha K(2 - 3K)) \mu^a \leq 0$ .

Finally, to establish (3.6) for the case in which  $K < 1/2$ , we can use the two inequalities (2.13 – 2.14) from Proposition 2.1 to get,

$$\begin{aligned} & (1 - \alpha K(2 - 3K)) \mu^a - (1 - \alpha K(1 - K)) \Gamma \\ \leq & (1 - \alpha K(2 - 3K)) \left( \Gamma - \frac{K}{2\phi(\Gamma)} \right) - (1 - \alpha K(1 - K)) \Gamma \\ = & -(1 - \alpha K(2 - 3K)) \frac{K}{2\phi(\Gamma)} - \alpha K(1 - 2K) \Gamma \\ = & \frac{K}{2\phi(\Gamma)} \cdot (-(1 - \alpha K(2 - 3K)) - 2\alpha(1 - 2K) \Gamma \phi(\Gamma)) \\ \leq & \frac{K}{2\phi(\Gamma)} \cdot (-(1 - \alpha K(2 - 3K)) + \alpha(1 - 2K)^2) \\ = & \frac{K}{2\phi(\Gamma)} \cdot (-(1 - \alpha) - \alpha K(2 - K)) < 0. \end{aligned}$$

■

The previous result shows, as in the previous section, that if crime increases, the social status loss associated with being arrested decreases. Therefore, as crime increases, the social disincentives diminish, inviting the possibility of multiple equilibrium. Our next result also confirms our earlier findings about the existence of multiple equilibria and that increasing the policy variables  $(\alpha, f)$  reduces crime at any stable equilibrium.

**Proposition 3.2.** *There may be multiple equilibria. Moreover, at any stable equilibrium, increasing the punishment,  $f$ , the probability of arrest,  $\alpha$ , or concern for social status,  $z$ , reduces crime.*

**Proof.** Individual  $\Gamma$  is, by definition, indifferent between stealing and working:

$$W \equiv \alpha(y + (1 - K)(f + S)) - g + \Gamma = 0.$$

In order to assess the effect of raising  $f$  on the criminal population, we write

$$\frac{d\Gamma}{df} = -\frac{W_f}{W_\Gamma}.$$

It is easy to show that  $W_f \geq 0$  such that increasing  $f$  causes  $W$  to become positive, making work more preferable than crime.

$$\begin{aligned} W_\Gamma &= 1 - \alpha [(f + S) K_\Gamma - (1 - K) S_\Gamma] \\ &= 1 - \alpha \left\{ \left( \frac{f}{z} - \frac{\mu^a}{1 - \alpha K (1 - K)} \right) \right. \\ &\quad \left. - (1 - K) \left( \frac{(1 - \alpha K (2 - 3K)) \mu^a - (1 - \alpha K (1 - K)) \Gamma}{K (1 - \alpha K (1 - K))^2} \right) \right\} \cdot \phi(\Gamma) z \end{aligned}$$

Since  $S_\Gamma \leq 0$ , the term in square brackets in the previous equation is positive. Therefore, there exists some distribution  $\phi$  sufficiently dense around the center and a sufficiently large concern for social status  $z > 0$  that ensure that  $W_\Gamma < 0$ . However, at any stable equilibrium,  $W_\Gamma > 0$ , and thus  $d\Gamma/df \leq 0$ . To obtain the results for  $\alpha$  and  $z$ , note that  $W_\alpha = y + (1 - K)(f - S) > 0$  and  $W_z = \alpha(1 - K)(\mu^n - \mu^a) > 0$ . ■

#### 4. Different ability groups

We have so far assumed that individuals differ only with respect to their psychic cost of engaging in crime,  $\gamma$ . Let us now generalize our discussion by considering also different ability types. In order to simplify the discussion, suppose that there are two ability types,  $y = y_H, y_L$ , where  $y_H > y_L > 0$ . Let  $r_i = 1 - r_j$  be the population proportion of ability type  $i = H, L$ . In our present setup, an individual each individual is then described by  $(i, \gamma)$ . Lastly, we assume that the psychic cost  $\gamma$  is distributed identically among the two groups and independently from  $y$ .

Since a criminal is assigned to a random individual from the population, her expected loot will now be

$$Y = r_H(1 - K_H)y_H + r_L(1 - K_L)y_L.$$

where the population proportion of criminals with ability  $i$  is defined as

$$K_i = \int_{-\infty}^{\Gamma_i} \phi(\gamma) d\gamma, \quad (4.1)$$



where  $\Gamma_i$  determines the threshold for group  $i = H, L$ . Also, the total population proportion of criminals is now given by

$$K = r_H K_H + r_L K_L.$$

Our discussion in this section allows us to distinguish between two types of social interaction mechanisms. In the previous sections, an individual's social status was dependent on how others perceived her preference type  $\gamma$ . Being perceived as an individual with low level of self-reproach was costly since such an event implied a social status loss. However, one may wonder if individuals within a particular ability group really care how they are perceived by individuals from another ability group. It is possible that ability types are observable, so that when an individual is arrested, her expected type can be determined by the proportion of criminals in her ability group. Both these assumptions lead us to distinguish between *intragroup interaction* and *intergroup interaction*. Let  $S_i$  and  $S_o$  denote the net expected status losses for the intragroup interaction for ability group  $i$  and intergroup interaction, respectively. We operationalize both types of interaction by assuming that the net expected status loss for any individual with ability  $i$  is given by  $\sigma_i$  as a convex combination of  $S_i$  and  $S_o$ :

$$\sigma_i = \beta S_i + (1 - \beta) S_o,$$

where  $\beta \in [0, 1]$ .

Let  $U_{0,i} = (1 - (1 - \alpha) K) y_i$  and  $U_1 = (1 - \alpha) Y$  be the expected incomes of  $(i, \gamma)$  from work and criminals activities, respectively. Then, individual  $(i, \gamma)$  will work if and only if her net utility from work,  $U_0 - g$ , exceeds her net utility from criminal activities,  $U_1 - \alpha (1 - K) (f + \sigma_i) - \gamma$ , or if  $W_i \geq 0$  where

$$W_i = U_{0,i} - g - U_1 + \alpha (1 - K) (f + \sigma_i) + \gamma. \quad (4.2)$$

When the social status is determined by intergroup interaction, the expected type for arrested criminals,  $\mu^a$ , is defined, as before, to be the expected type of criminals. However, we now consider two groups of individuals. The probability that an arrested individual has ability  $i$  is simply the proportion of criminals with that ability among all criminals,  $r_i K_i / K$ . The expected preference type of a potential criminal with ability  $i$  is defined by

$$\mu_i^a = \frac{1}{K_i} \cdot \int_{-\infty}^{\Gamma_i} \gamma \phi(\gamma) d\gamma.$$

Combining the two ability types, we may now write the expected type of a potential criminal as

$$\mu_o^a = \frac{1}{K} \cdot \sum_{i=H,L} r_i K_i \mu_i^a = \frac{1}{K} \cdot \left( \sum_{i=H,L} r_i \int_{-\infty}^{\Gamma_i} \gamma \phi(\gamma) d\gamma \right). \quad (4.3)$$

Similarly, the expected type of individuals who are not arrested,  $\mu^n$ , can now be written as a function of the expected types for both ability groups. However, remember from our discussion in the previous section that individuals who are not arrested can either be workers or criminals who were either unable to victimize a worker or avoided being captured. Note, however, that unlike before we will need to consider the expected type of workers and criminals within each ability group, just like we did for the calculation of  $\mu_o^a$  above. The probability that an individual who is not convicted is from ability group  $i$  is equivalent to the proportion of ability  $i$  individuals who were not arrested,  $1 - \alpha K_i (1 - K)$ , divided by the population proportion of non-convicted individuals,  $1 - \alpha K (1 - K)$ . The probability that a random pick from the population of free individuals is a criminal with ability  $i$  is equivalent to the proportion of ability  $i$  criminals who were not arrested,  $K_i (1 - \alpha (1 - K))$ , divided by the population proportion of individuals who were not arrested,  $1 - \alpha K (1 - K)$ . Since the entire population is comprised of both type  $H$  and type  $L$  individuals, the expected type of an individual who was not arrested,  $\mu_o^n$ , will be determined by a weighted combination of proportions of criminals and workers from both groups. More succinctly, the expected type of an individual who was not arrested can now be simplified to

$$\begin{aligned} \mu_o^n &= \frac{1 - \alpha(1 - K)}{1 - \alpha K(1 - K)} \cdot \left( \sum_{i=H,L} r_i \int_{-\infty}^{\Gamma_i} \gamma \phi(\gamma) d\gamma \right) \\ &\quad + \frac{1}{1 - \alpha K(1 - K)} \cdot \left( \sum_{i=H,L} r_i \int_{\Gamma_i}^{+\infty} \gamma \phi(\gamma) d\gamma \right) \\ &= -\frac{\alpha K(1 - K)}{1 - \alpha K(1 - K)} \cdot \mu_o^a. \end{aligned} \quad (4.4)$$

After rearranging the terms, we find that the net expected social status loss for the intergroup interaction model,  $S_o$ , can be simplified to the familiar form

$$S_o = (\mu_o^n - \mu_o^a) \cdot z = -\frac{\mu_o^a}{1 - \alpha K(1 - K)} \cdot z. \quad (4.5)$$

**Lemma 4.1.** For any  $i, j \in \{H, L\}$  with  $\Gamma_i \geq \Gamma_j$ , as the threshold for group  $i$  increases, the net intergroup expected status loss decreases, or  $\partial S_o / \partial \Gamma_i < 0$ .

**Proof.** The derivative of the expected type of a criminal is

$$\frac{\partial \mu_o^a}{\partial \Gamma_i} = r_i \cdot \frac{\Gamma_i - \mu_o^a}{K} \cdot \phi(\Gamma_i).$$

The previous term is strictly positive for  $i$  but may be negative for  $j$ . Now, we have

$$\begin{aligned} \frac{\partial S_o}{\partial \Gamma_i} &= - \frac{\frac{\partial \mu_o^a}{\partial \Gamma_i} (1 - \alpha K (1 - K)) + \alpha r_i (1 - 2K \cdot K_i) \mu_o^a \phi(\Gamma_i)}{(1 - \alpha K (1 - K))^2} \cdot z \\ &= - \frac{\frac{\Gamma_i - \mu_o^a}{K} (1 - \alpha K (1 - K)) + \alpha (1 - 2K \cdot K_i) \mu_o^a}{(1 - \alpha K (1 - K))^2} \cdot r_i z \phi(\Gamma_i) \\ &= - \frac{(\Gamma_i - \mu_o^a) (1 - \alpha K (1 - K)) + \alpha K (1 - 2K \cdot K_i) \mu_o^a}{K (1 - \alpha K (1 - K))^2} \cdot r_i z \phi(\Gamma_i) \\ &= - \frac{(1 - \alpha K (1 - K)) \Gamma_i - (1 - \alpha K (2(1 - K K_i) - K)) \mu_o^a}{K (1 - \alpha K (1 - K))^2} \cdot r_i z \phi(\Gamma_i) < 0. \end{aligned}$$

The last inequality follows from the fact that

$$(1 - \alpha K (2(1 - K K_i) - K)) \mu_o^a - (1 - \alpha K (1 - K)) \Gamma_i < 0 \quad (4.6)$$

To see this, note that when  $K = 1/2$ , the inequality is a direct consequence of  $\mu_o^a < \Gamma_i$ . When  $K > 1/2$ ,  $\Gamma_i > 0$  and therefore  $(1 - \alpha K (1 - K)) \Gamma_i > 0$  while  $(1 - \alpha K (2 - 3K)) \mu_o^a < 0$ .

Finally, to establish (4.6) for the case in which  $K < 1/2$ , note first that since  $\Gamma_i \geq \Gamma_j$ , we have  $-K_i \leq -K \leq -K_j$  and  $\mu_i^a \geq \mu_o^a \geq \mu_j^a$ . Then, we write the following two inequalities, which are similar to their counterparts in equations (2.13 – 2.14) in Proposition 2.1:

$$\mu_o^a \leq \mu_i^a \leq -\frac{K_i}{2\phi(\Gamma_i)} + \Gamma_i \leq -\frac{K}{2\phi(\Gamma_i)} + \Gamma_i$$

and

$$-\Gamma_i \phi(\Gamma_i) \leq \frac{1}{2} - K_i = \frac{1 - 2K_i}{2} \leq \frac{1 - 2K}{2}$$

Using the properties just outlined, we can establish the inequality in (4.6) when  $K < 1/2$ :

$$\begin{aligned}
& (1 - \alpha K (2(1 - K K_i) - K)) \mu_o^a - (1 - \alpha K (1 - K)) \Gamma_i \\
\leq & (1 - \alpha K (2 - 3K)) \mu_o^a - (1 - \alpha K (1 - K)) \Gamma_i \\
\leq & (1 - \alpha K (2 - 3K)) \mu_i^a - (1 - \alpha K (1 - K)) \Gamma_i \\
\leq & (1 - \alpha K (2 - 3K)) \left( \Gamma_i - \frac{K}{2\phi(\Gamma_i)} \right) - (1 - \alpha K (1 - K)) \Gamma_i \\
= & -(1 - \alpha K (2 - 3K)) \frac{K}{2\phi(\Gamma_i)} - \alpha K (1 - 2K) \Gamma_i \\
= & \frac{K}{2\phi(\Gamma_i)} \cdot (-(1 - \alpha K (2 - 3K)) - 2\alpha (1 - 2K) \Gamma_i \phi(\Gamma_i)) \\
\leq & \frac{K}{2\phi(\Gamma_i)} \cdot (-(1 - \alpha K (2 - 3K)) + \alpha (1 - 2K)^2) \\
= & \frac{K}{2\phi(\Gamma_i)} \cdot (-(1 - \alpha) - \alpha K (2 - K)) < 0. \blacksquare
\end{aligned}$$

We now introduce the intragroup interaction mechanism, in which concern for status is determined within a group. Suppose that we pick a random agent from the population of individuals who are not arrested. As was discussed previously, that agent may be a criminal or a worker. In particular, the probability of picking a criminal from all ability  $i$  individuals is simply the proportion of ability  $i$  criminals who were either not able to victimize a worker or who were not caught,  $K_i (1 - \alpha (1 - K))$ , divided by the population proportion of ability  $i$  individuals who were not caught,  $1 - \alpha K_i (1 - K)$ .<sup>9</sup> For the same ability group, the probability of picking a worker, then, is simply the proportion of ability  $i$  workers,  $1 - K_i$ , divided again by the population proportion of ability  $i$  individuals who were not caught. The expected (preference) type of an ability  $i$  individual who was not arrested is

$$\begin{aligned}
\mu_i^n &= \frac{K_i (1 - \alpha (1 - K))}{1 - \alpha K_i (1 - K)} \cdot \frac{\int_{-\infty}^{\Gamma_i} \gamma \phi(\gamma) d\gamma}{K_i} \\
&+ \frac{1 - K_i}{1 - \alpha K_i (1 - K)} \cdot \frac{\int_{\Gamma_i}^{+\infty} \gamma \phi(\gamma) d\gamma}{1 - K_i} \\
&= -\frac{\alpha K_i (1 - K)}{1 - \alpha K_i (1 - K)} \cdot \mu_i^a
\end{aligned}$$

---

<sup>9</sup>Note that  $1 - K$  still gives the probability that a criminal will be assigned to a worker.

The expected (net) social loss in the event of being arrested for the intragroup interaction model is then given by

$$S_i = (\mu_i^n - \mu_i^a) \cdot z = -\frac{\mu_i^a}{1 - \alpha K_i (1 - K)} \cdot z. \quad (4.7)$$

Our next result shows that increasing crime reduces the net expected social status loss for the intragroup interaction model.

**Lemma 4.2.** *For any  $i, j = H, L$ , as threshold  $\Gamma_j$  increases, the net intragroup expected status loss for group  $i$  decreases, or  $\partial S_i / \partial \Gamma_j < 0$ .*

**Proof.** First, we can easily show that

$$\frac{\partial \mu_i^a}{\partial \Gamma_i} = \frac{\Gamma_i - \mu_i^a}{K_i} \cdot \phi(\Gamma_i) > 0.$$

Also, note that  $\partial \mu_i^a / \partial \Gamma_j = 0$  for  $j \neq i$ . Now, we have

$$\begin{aligned} \frac{\partial S_i}{\partial \Gamma_i} &= -\frac{\frac{\partial \mu_i^a}{\partial \Gamma_i} (1 - \alpha K_i (1 - K)) + \alpha (1 - K - r_i K_i) \mu_i^a \phi(\Gamma_i)}{(1 - \alpha K_i (1 - K))^2} \cdot z \\ &= -\frac{\left(\frac{\Gamma_i - \mu_i^a}{K_i}\right) (1 - \alpha K_i (1 - K)) + \alpha (1 - K - r_i K_i) \mu_i^a}{(1 - \alpha K_i (1 - K))^2} \cdot \phi(\Gamma_i) z \\ &= -\frac{(\Gamma_i - \mu_i^a) (1 - \alpha K_i (1 - K)) + \alpha K_i (1 - K - r_i K_i) \mu_i^a}{K_i (1 - \alpha K_i (1 - K))^2} \cdot \phi(\Gamma_i) z. \\ &= \frac{(1 - \alpha K_i (2(1 - K) - r_i K_i)) \mu_i^a - (1 - \alpha K_i (1 - K)) \Gamma_i}{K_i (1 - \alpha K_i (1 - K))^2} \cdot \phi(\Gamma_i) z < 0. \end{aligned}$$

The last inequality follows from the fact that

$$(1 - \alpha K_i (2(1 - K) - r_i K_i)) \mu_i^a - (1 - \alpha K_i (1 - K)) \Gamma_i < 0 \quad (4.8)$$

To see this, note that when  $K_i = 1/2$ , the inequality is a direct consequence of  $\mu_i^a < \Gamma_i$ . When  $K_i > 1/2$ ,  $\Gamma_i > 0$  and therefore  $(1 - \alpha K_i (1 - K)) \Gamma_i > 0$  while  $(1 - \alpha K_i (2(1 - K) - r_i K_i)) \mu_i^a < 0$ .

To establish (3.6) for when  $K_i < 1/2$ , we use the following inequalities already outlined in the proof of Lemma 4.1:

$$\mu_i^a \leq -\frac{K_i}{2\phi(\Gamma_i)} + \Gamma_i$$

and

$$-\Gamma_i \phi(\Gamma_i) \leq \frac{1}{2} - K_i = \frac{1 - 2K_i}{2}$$

Using the properties, we can establish the inequality in (4.8) when  $K_i < 1/2$  :

$$\begin{aligned} & (1 - \alpha K_i (2(1 - K) - r_i K_i)) \mu_i^a - (1 - \alpha K_i (1 - K)) \Gamma_i \\ \leq & (1 - \alpha K_i (2(1 - K) - r_i K_i)) \left( -\frac{K_i}{2\phi(\Gamma_i)} + \Gamma_i \right) - (1 - \alpha K_i (1 - K)) \Gamma_i \\ = & -(1 - \alpha K_i (2(1 - K) - r_i K_i)) \frac{K_i}{2\phi(\Gamma_i)} - \alpha K_i (1 - K - r_i K_i) \Gamma_i \\ = & \frac{K_i}{2\phi(\Gamma_i)} \cdot (-(1 - \alpha K_i (2(1 - K) - r_i K_i)) - 2\alpha (1 - K - r_i K_i) \Gamma_i \phi(\Gamma_i)) \\ \leq & \frac{K}{2\phi(\Gamma_i)} \cdot (-(1 - \alpha K_i (2(1 - K) - r_i K_i)) + \alpha (1 - K - r_i K_i) (1 - 2K_i)) \\ = & \frac{K}{2\phi(\Gamma_i)} \cdot (-(1 - \alpha) - \alpha r_i K_i (1 - K_i) - \alpha K) < 0. \end{aligned}$$

Finally, when  $j \neq i$ , we can simply see that

$$\frac{\partial S_i}{\partial \Gamma_j} = \frac{\alpha r_j K_i \mu_i^a}{1 - \alpha K_i (1 - K)} < 0. \blacksquare$$

Since we have two ability groups, we have to modify our stability results. In particular, our stability needs to ensure that perturbations will not start an adjustment process away from the original equilibrium. Let

$$\Gamma(t) = \begin{bmatrix} \Gamma^H(t) \\ \Gamma^L(t) \end{bmatrix}$$

denote the two thresholds at any point in time  $t \geq 0$ . As before, we let the equilibrium dynamics be governed by the non-linear function

$$\dot{\Gamma}(t) = D_t \Gamma(t) \equiv -K \cdot W(\Gamma(t)), \quad (4.9)$$

where  $K$  is a diagonal matrix whose diagonal elements,  $k_i$ , determine the speed of adjustment for group  $i = H, L$ .

We now determine the conditions that guarantee that  $\Gamma$  is stable. Our next result utilizes the decision functions  $W_H$  and  $W_L$ , as defined in equation (4.2).

**Lemma 4.3.** *Let  $A$  be the Jacobian matrix for  $W_H$  and  $W_L$ , such that*

$$A = \begin{bmatrix} a_{HH} & a_{HL} \\ a_{LH} & a_{LL} \end{bmatrix}$$

*where  $a_{ij} = dW_i/d\Gamma_j$  for any  $i, j \in \{H, L\}$ . Then,  $\Gamma$  is locally stable if*

$$\det A = a_{HH} \cdot a_{LL} - a_{HL} \cdot a_{LH} > 0 \text{ and } a_{HH} + a_{LL} > 0.$$

**Proof.** The *local* stability of an equilibrium  $\Gamma$  is guided by a linear approximation of (4.9), or by

$$\dot{\Gamma}(t) = -K \cdot A \cdot [\Gamma(t) - \Gamma].$$

We now apply the Routh-Hurwitz condition to determine the local stability properties for some  $\Gamma$ . The characteristic equation for  $-A$  is given by

$$\lambda^2 - (a_{HH} + a_{LL}) \lambda + \det A. \quad (4.10)$$

Then, the Routh-Hurwitz condition implies that both  $a_{HH} + a_{LL} > 0$  and  $\det A > 0$  have to be true in order to have  $\Gamma$  as a locally stable equilibrium. ■

Our next result shows that when the social status is determined by intergroup interaction, then the low ability group has more crime than the high ability group. This result is quite easy to interpret: Since criminals have access to both groups for their criminal activities, increasing the income differential between the two groups naturally leads to more crime in the low ability group. However, the same result can not be obtained if social status is determined by intragroup interaction.

**Proposition 4.4.** *The following results apply to stable equilibria.*

- i). When social status is determined by intergroup interaction,  $\beta = 0$ , increasing punishment  $f$  reduces crime for the high ability group, but may increase crime for the low ability group if the income dispersion,  $y_H - y_L$ , is sufficiently large. When  $\beta > 0$ , increasing punishment  $f$  may increase crime for either group.*
- ii). When  $\beta = 0$ , there is less crime in the high ability group, i.e.  $K_H < K_L$ . However, if  $\beta > 0$ , the high ability group may have more crime than the low ability group.*
- iii). Increasing the intragroup interaction factor  $\beta$  may increase, decrease, or leave aggregate crime unchanged.*

**Proof.** Before proving the results, we identify the properties of the elements of the Jacobian matrix  $A$ . Then, we will use the Implicit Function theorem to obtain local comparative statics.

The elements of  $A$  are given by

$$\begin{aligned} a_{ii} &= \frac{dW_i}{d\Gamma_i} = 1 - \alpha \left( r_i (f + \sigma_i) \cdot \phi(\Gamma_i) - (1 - K) \cdot \frac{\partial \sigma_i}{\partial \Gamma_i} \right) \\ a_{ij} &= \frac{dW_i}{d\Gamma_j} = - (1 - \alpha) (1 - r_i) (y_i - y_j) \cdot \phi(\Gamma_j) \\ &\quad - \alpha \left( r_j (f + \sigma_i) \cdot \phi(\Gamma_j) - (1 - K) \cdot \frac{\partial \sigma_i}{\partial \Gamma_j} \right). \end{aligned}$$

We first show the sign of  $a_{HL} - a_{LL}$ . Note that by omitting the common terms of  $a_{HL}$  and  $a_{LL}$ , we can write

$$\begin{aligned} a_{HL} - a_{LL} &= -1 - (1 - \alpha) (1 - r_H) (y_H - y_L) \cdot \phi(\Gamma_L) \\ &\quad - \alpha \beta \left( r_L (S_H - S_L) \phi(\Gamma_L) - (1 - K) \left( \frac{\partial S_H}{\partial \Gamma_L} - \frac{\partial S_L}{\partial \Gamma_L} \right) \right) \end{aligned} \quad (4.11)$$

It is easy to see that if  $\beta = 0$ , then  $a_{HL} - a_{LL} < 0$ .

We now show that when  $\beta > 0$  and  $\alpha > 0$ , it is possible to have  $a_{HL} - a_{LL} \geq 0$ . Since  $\mu_H^a < 0$  by definition, we know that

$$\begin{aligned} & r_L \cdot S_H \cdot \phi(\Gamma_L) - (1 - K) \cdot \frac{\partial S_H}{\partial \Gamma_L} \\ &= - \frac{r_L \mu_H^a}{1 - \alpha K_H (1 - K)} \cdot \phi(\Gamma_L) z \\ &\quad - \frac{\alpha K_H (1 - K) r_L \mu_H^a}{1 - \alpha K_H (1 - K)} \cdot \phi(\Gamma_L) z \\ &= - \frac{1 + \alpha K_H (1 - K)}{1 - \alpha K_H (1 - K)} \cdot r_L \mu_H^a \cdot \phi(\Gamma_L) z > 0 \end{aligned}$$

Similarly, we can show that

$$\begin{aligned} 0 &< (1 - K) \cdot \frac{\partial S_L}{\partial \Gamma_L} - r_L \cdot S_L \cdot \phi(\Gamma_L) \\ &= \frac{(1 - \alpha K_L (2(1 - K) - r_L K_L)) \mu_L^a - (1 - \alpha K_L (1 - K)) \Gamma_L}{K_L (1 - \alpha K_L (1 - K))^2} \cdot (1 - K) \phi(\Gamma_L) z \end{aligned}$$



$$\begin{aligned}
& + \frac{r_L \mu_L^a}{1 - \alpha K_L (1 - K)} \cdot \phi(\Gamma_L) z \\
= & \frac{((1 - K)(1 - \alpha K_L (2(1 - K) - r_L K_L)) + r_L K_L (1 - \alpha K_L (1 - K))) \mu_L^a}{K_L (1 - \alpha K_L (1 - K))^2} \cdot \phi(\Gamma_L) z \\
& - \frac{\Gamma_L}{K_L (1 - \alpha K_L (1 - K))} \cdot (1 - K) \phi(\Gamma_L) z \\
= & \frac{(1 - 2\alpha K_L (1 - K(2 - K)) - K + r_L K_L)}{K_L (1 - \alpha K_L (1 - K))^2} \cdot \mu_L^a \cdot \phi(\Gamma_L) z \\
& - \frac{(1 - K) \Gamma_L}{K_L (1 - \alpha K_L (1 - K))} \cdot \phi(\Gamma_L) z \\
< & 0
\end{aligned}$$

where the inequality in the beginning of the previous arguments follows since  $\partial S_L / \partial \Gamma_L < 0$  by Lemma 4.2 and  $S_L \geq 0$ . To illustrate that the term in brackets in equation (4.11) could be positive, let  $\Gamma = \Gamma_H = \Gamma_L = 0$ , which enables us to let  $\mu \equiv \mu_H^a = \mu_L^a$  and  $K = K_H = K_L = 1/2$ . In that case, we have

$$\begin{aligned}
& r_L (S_H - S_L) \phi(0) - (1 - K) \left( \frac{\partial S_H}{\partial \Gamma_L} - \frac{\partial S_L}{\partial \Gamma_L} \right) \\
= & \frac{16 - 8\alpha + r\alpha^2}{(4 - \alpha)^2} \cdot \mu \cdot \phi(0) z < 0.
\end{aligned}$$

Therefore,  $a_{HL} - a_{LL} \geq 0$  could be true for some distribution  $\phi$  that is sufficiently dense around the center and a sufficiently large concern for social status  $z > 0$ .

We now obtain the sign for  $a_{LH} - a_{HH}$ . After omitting common terms, we have

$$\begin{aligned}
a_{HL} - a_{LL} = & -1 - (1 - \alpha)(1 - r_L)(y_L - y_H) \cdot \phi(\Gamma_H) \\
& - \alpha \beta \left( r_H (S_L - S_H) \phi(\Gamma_H) - (1 - K) \left( \frac{\partial S_L}{\partial \Gamma_H} - \frac{\partial S_H}{\partial \Gamma_H} \right) \right).
\end{aligned}$$

Note that when  $\beta = 0$ , if  $y_L - y_H$  is sufficiently large, then it is possible that  $a_{HL} - a_{LL} \geq 0$ . When  $\beta > 0$ , we can use a similar argument as above to show that the same  $a_{HL} - a_{LL} \geq 0$  is also possible.

We are now ready to prove our first result. The Implicit Function theorem implies that

$$-A^{-1} \cdot D_f W = D_f \Gamma$$

where

$$D_f W = \begin{bmatrix} \frac{\partial W_H}{\partial f} \\ \frac{\partial W_L}{\partial f} \end{bmatrix} = \begin{bmatrix} \alpha(1-K) \\ \alpha(1-K) \end{bmatrix},$$

and

$$D_f \Gamma = \begin{bmatrix} \frac{\partial \Gamma_H}{\partial f} \\ \frac{\partial \Gamma_L}{\partial f} \end{bmatrix}.$$

By carrying out these operations, we get

$$\begin{bmatrix} \frac{\partial \Gamma_H}{\partial f} \\ \frac{\partial \Gamma_L}{\partial f} \end{bmatrix} = \begin{bmatrix} \frac{\alpha(1-K)(a_{HL}-a_{LL})}{\det A} \\ \frac{\alpha(1-K)(a_{LH}-a_{HH})}{\det A} \end{bmatrix}.$$

Then, when punishment is increased,  $\Gamma_i$  decreases only when  $a_{ij} - a_{jj} < 0$ .

In order to obtain our second result, we simply subtract  $W_L$  from  $W_H$  in order to omit common variables:

$$\begin{aligned} W_H - W_L &= (1 - \alpha(1-K))(y_H - y_L) \\ &\quad + \alpha\beta(1-K)(S_H - S_L) + \Gamma_H - \Gamma_L. \end{aligned}$$

It is quite easy to see that  $\beta = 0$  and  $y_H - y_L > 0$  imply that  $\Gamma_H - \Gamma_L < 0$ . However, when  $\beta > 0$ , it is possible to have  $\Gamma_H - \Gamma_L > 0$ . To see this, note that  $S_H - S_L$  and  $\Gamma_H - \Gamma_L$  have opposite signs by lemma 4.2. Also, note that neither of the stability conditions restrict the magnitude or sign of  $S_H - S_L$ . In particular,  $S_H$  and  $S_L$  enter the two requirements  $\det A > 0$  and  $a_{HH} + a_{LL} > 0$  multiplicatively and additively, respectively.

In order to obtain our third result, note first that

$$\begin{aligned} \xi_H &\equiv \frac{\partial W_H}{\partial \beta} = \alpha(1-K)(S_H - S_o) \\ \xi_L &\equiv \frac{\partial W_L}{\partial \beta} = \alpha(1-K)(S_L - S_o) \end{aligned}$$

Based on our second result, we only know that  $\xi_H$  and  $\xi_L$  have opposite signs. We can now write

$$\begin{aligned} \begin{bmatrix} \frac{\partial \Gamma_H}{\partial y_L} \\ \frac{\partial \Gamma_L}{\partial y_L} \end{bmatrix} &= - \begin{bmatrix} a_{HH} & a_{HL} \\ a_{LH} & a_{LL} \end{bmatrix}^{-1} \cdot \begin{bmatrix} \xi_H \\ \xi_L \end{bmatrix} \\ &= \begin{bmatrix} \frac{a_{HL} \cdot \xi_L - a_{LL} \cdot \xi_H}{\det A} \\ \frac{a_{LH} \cdot \xi_H - a_{HH} \cdot \xi_L}{\det A} \end{bmatrix}. \end{aligned}$$

The results obtained above show that  $a_{ij} - a_{jj} < 0$  for all  $i = H, L$  and  $j \neq i$ . Since  $\xi_H$  and  $\xi_L$  are of opposite signs and since  $a_{ij} - a_{jj}$  has an ambiguous sign, we can not use these results to determine how the individuals respond to increased intergroup interaction. ■

## 5. Conclusion

In this paper, we considered a large population of individuals who choose between engaging in crime and working. When individuals are concerned with their social status, we find that increasing crime will, in most cases, reduce the net social status loss associated with being arrested. This induces the possibility of multiple equilibria, even within a simple activity choice model. Also, we find that in stable equilibria increasing punishment reduces crime when the all individuals have the same ability. Moreover, when the society is comprised of two ability groups, whether social status is determined within a group or among the two groups causes our results to change. In particular, unlike earlier models, we show that it is possible that increasing punishment may increase crime. It is also possible for the higher earning ability group to have more crime than the low ability group, if social status is determined within a group.

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